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13. ABSTRACT *(Maximum 200 words)*
Research is reported primarily in the following areas: [1] High-frequency enhancements of the backscattering of sound by tilted (empty and water-filled) blunt elastic cylindrical shells in water are calculated and observed. The leaky-wave contributions to backscattering were studied and were also studied for a solid stainless-steel cylinder. A different type of backscattering enhancement was also studied for penetrable cylinders (e.g. plastics) supporting low-velocity waves. This enhancement is associated with a caustic-merging transition which was also confirmed with an analogous light scattering experiment. [2] The coupling of oscillating magnetic fields to the torsional modes of a stainless steel spherical shell in water was investigated. (Sound radiated by the magnetically excited modes is detected and resonances are identified.) [3] Some predicted properties of helicoidal ultrasonic waves were confirmed using a novel PVDF transducer. These waves have an axial null and have potential applications in scattering and alignment and they carry orbital angular momentum. Other research summarized concerns [4] acoustical-scattering analogs studied with light and [5] the acoustical probing of dilute aqueous suspensions of particles using radiation pressure.

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GEOMETRICAL ASPECTS OF SCATTERING AND PHYSICAL EFFECTS OF SOUND

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June 1998

Annual Summary Report for Grant N00014-92-J-1600 and AASERT Grant N00014-97-1-0614

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The project description, approaches, and summary of accomplishments are grouped according to Projects I, II, III, IV, and V.

I. A. Project Description: Transient and High-Frequency Enhancements in Scattering from Elastic Objects -- The objective is to identify and model physical processes that are likely to be important for the classification or detection of scatterers in water based on high frequency scattering signatures and acoustical images. When possible, quantitative ray approximations are formulated to predict scattering amplitudes and experiments are used to test the approximations. The task emphasis this year has been on scattering by tilted bluntly-truncated elastic shells and solid cylinders.

B. Approach: *(i) Theoretical*: The emphasis was on extending Marston's ray formulation for approximating large leaky wave contributions to backscattering to situations not previously considered. Three publications describing the formulation and applications previously considered appeared during the year [JASA 102, 358-369 and 1628-1638 (1997) and 23rd Symposium on Acoustical Imaging, pp. 369-374]. In addition, the AASERT student (F. Blonigen) calculated backscattering enhancements for solid cylinders made of low-wave-velocity materials such as plastics. *(ii) Experimental*: Backscattering was measured as a function of tilt angle for empty and water-filled cylindrical shells (S. Morse) and a solid stainless steel cylinder (K. Gipson). Solid cylinders made of low-wave-velocity materials were measured over a selected range of angles (F. Blonigen). *(iii) Computational*: With assistance from G. Kaduchak (UT:ARL), Morse has been applying an approximate computational model applicable to thick finite tilted cylinders.

C. Accomplishments:

- *Marston:* The principal result was the extension of the analysis of the magnitude of the meridional ray backscattering amplitude for tilted finite cylinders [as described in P. L. Marston, JASA 102, 358-369 (1997)] to situations where the tilt angle of the cylinder differs from the leaky wave coupling angle $\theta_l = \sin^{-1}(c/c_l)$.
- *Gipson:* In her completed Ph.D. thesis, Karen Gipson compared the result of the calculation noted above with measurements of tone-burst backscattering amplitudes as a function of tilt . The measurements were for meridional leaky Rayleigh waves on a solid stainless steel cylinder in water. Some other backscattering enhancements with this cylinder were measured and analyzed. An important example is the enhancement resulting from a ray which runs across the diameter of the end of the cylinder and reflects from the

rim. Gipson's thesis also extends the analysis of our earlier experiments on the backscattering enhancement for leaky Rayleigh waves on tilted cubes.

- *Morse:* In the continuation of his Ph.D. thesis research and writing, Scot Morse has made several new quantitative (as well as qualitative) measurements and calculations of meridional ray backscattering enhancements from finite cylindrical shells in water. The previous work demonstrates that even at high frequencies, bluntly-truncated thick (as well as moderately thin) tilted cylindrical shells can have much larger backscattering than for a rigid cylinder of the same shape over a wide angular range [Morse, Marston, and Kaduchak, JASA 103, 785-794 (1998)]. The new work gives quantitative tests of our theory describing the angle and frequency dependence. The comparisons are with amplitude measurements and, in certain cases, with an approximate partial-wave series computational model. Morse has also investigated (i) the time-frequency display of the impulse response, (ii) elastic contributions to synthetic aperture images, and (iii) the frequency dependence associated with the mechanics of leaky wave reflection from the ends of the cylinder.
- *Appendices giving a more detailed summary:* Research by Marston, Gipson, and Morse is summarized in greater detail in Appendices A and B of this report. These items are the extended abstracts submitted to the *16th International Congress on Acoustics* (Seattle, 1998).
- *Blonigen:* The AASERT supported student (F. Blonigen) for part of the year confirmed the existence of a new kind of ultrasonic backscattering enhancement for tilted solid cylinders made of materials having low elastic wave velocities (e.g. polystyrene). The enhancement is the result of a caustic merging transition that occurs in a critical range of tilted angles for the Airy (or rainbow) caustic produced by refraction and reflection at the solid-water interface of the cylinder [F. J. Blonigen and P. L. Marston (abstract) JASA 102, 3088 (1997)]. Blonigen also developed a theory for this enhancement and has been working on the theory for other backscattering enhancements.

II. A. Project Description: Interaction of Sound with Sound Mediated by Suspensions of Particles -- The objective of this project is to understand how suspended particles affect the interaction of sound with sound in water. Large acoustic signatures are observed even with very small volume fractions of suspended particles. A summary of our early work recently appeared [H. J. Simpson and P. L. Marston, in *Nonlinear Acoustics* (Academic Press, 1998) pp. 399-420].

B. Approach: The emphasis of the research during the past year was on extending Kwiatkowski's analysis of the shift in the resonance frequency of an ultrasonic standing

wave resonator. The frequency shift is associated with the migration of particles due to the radiation pressure of the standing wave. A more general expression for the radiation force on a particle in an axisymmetric standing wave was also derived.

C. Accomplishments:
The detuning based on adiabatic invariance agrees with the general magnitude of the observed shift from Kwiatkowski's 1997 Ph.D. thesis. This work has been accepted for publication [C. S. Kwiatkowski and P. L. Marston, JASA (at press)].

III. A. Project Description: Electromagnetic Excitation of Elastic Modes of a Shell -- The objective is to use oscillating magnetic fields to excite specific modes of electrically conducting shells so as to produce easily detectable acoustic radiation in the surrounding fluid. This has potential application to the spectroscopy of fluid-loaded shell modes only weakly coupled to sound waves.

B. Approach: The emphasis was on driving torsional modes of a thick stainless steel spherical shell in water.

C. Accomplishments: B. T. Hefner extended his early measurements of the low-lying high-Q torsional modes of the spherical shell [B. T. Hefner and P. L. Marston (abstract) JASA 102, 3131 (1997)]. The modes were acoustically detected with a hydrophone.

IV. A. Project Description: Helicoidal Mode Transduction and Acoustic Holography -- This project concerns the transduction of a new form of acoustic beam in water and the diagnostics of our novel transducer using acoustic holography. Our ability to carry out high-frequency holography in water with a scanned hydrophone is likely to carry over into other applications.

B. Approach: Helicoidal waves are described by solutions to the Helmholtz equation having a vanishing amplitude along an axis of symmetry. Previous optics research pertaining to this solution and potential applications in acoustics are reviewed in Appendix C of this report which is an extended abstract for the *16th International Congress on Acoustics* (Seattle, 1998). Applications include scattering, alignment, and angular momentum transport.

C. Accomplishments: B. T. Hefner demonstrated one method for the approximate synthesis of ultrasonic helicoidal waves in water as described in Appendix C. In addition, he found that back-propagation of scanned hydrophone measurements obtained in a plane were useful for evaluating the operation of the transducer.

V. A. Project Description: Light Scattering by Dielectric Spheroids and Tilted Cylinders Relevant to Acoustic Analogs

B. Approach: Scattering patterns resulting from refraction and internal reflection for certain penetrable shapes may be more easily explored and recorded with light than with sound. The patterns are recorded and interpreted using extensions of ray theory for cases giving significant scattering enhancements.

C. Accomplishments:

- A novel analysis of the caustic-merging-transition in rainbow caustics from a tilted dielectric cylinder was published [Marston, Appl. Opt. 37, 1551-1556 (1998)].
- Measurements which confirm the aforementioned analysis were published [Mount, Thiessen, and Marston, Appl. Opt. 37, 1534-1539 (1998)], and this work was the basis of Catherine Mount's M.S. thesis (1998).
- Our earlier discovery of higher-order caustics in the scattering patterns for rays making three internal reflections within an oblate drop was analyzed and published [Langley and Marston, Appl. Opt. 37, 1520-1536 (1998)].
- By invitation from an editor of *Nature*, Marston published a *News and Views* discussion of research on the time-resolved electromagnetic terahertz glory of dielectric spheres [Marston, Nature 391, 841-842 (1998)].

OFFICE OF NAVAL RESEARCH
PUBLICATIONS/PATENTS/PRESENTATIONS/HONORS REPORT
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01 June 97 through 31 May 98

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a. Number of papers submitted to refereed journals but not yet published:	1
b. Number of papers published in refereed journals (ATTACH LIST):	7
c. Number of books or chapters submitted but not yet published:	1
d. Number of books or chapters published (ATTACH LIST):	2
e. Number of printed technical reports & non-refereed papers (ATTACH LIST):	5
f. Number of patents filed:	0
g. Number of patents granted (ATTACH LIST):	0
h. Number of invited presentations at workshops or professional society meetings:	1
i. Number of contributed presentations at workshops or professional society meetings:	10
j. Honors/awards/prizes for contract/grant employees, such as scientific society and faculty awards/offices (ATTACH LIST):	0
k. Number of graduate students supported at least 25% this year this contract/grant:	5
l. Number of post docs supported at least 25% this year this contract/grant:	0

How many of each are females or minorities? These six numbers are for ONR's EEO/Minority Reports. Minorities include Blacks, Aleuts, Amindians, etc., and those of Hispanic or Asian extraction/nationality. The Asians are singled out to facilitate meeting reporting semantics re "underrepresented".

Graduate student FEMALE:	2	Post doc FEMALE:	
Graduate student MINORITY:	1	Post doc MINORITY:	
Graduate student ASIAN E/N:		Post doc ASIAN:	

b. Papers Published in Refereed Journals:

1. P. L. Marston, "Approximate meridional leaky ray amplitudes for tilted cylinders: End-backscattering enhancements and comparisons with exact theory for infinite solid cylinders," J. Acoust. Soc. Am. 102, 358-369 (1997). Erratum: J. Acoust. Soc. Am. 103, 2236 (1998).

2. P. L. Marston, "Spatial approximation of leaky wave surface amplitudes for three-dimensional high-frequency scattering: Fresnel patches and application to edge-excited and regular helical waves on cylinders," J. Acoust. Soc. Am. 102, 1628-1638 (1997).

3. S. F. Morse, P. L. Marston, and G. Kaduchak, "High frequency backscattering enhancements by thick finite cylindrical shells in water at oblique incidence: Experiments, interpretation and calculations," J. Acoust. Soc. Am. 103, 785-794 (1998). [Kaduchak supported by UT:ARL.]

4. P. L. Marston, "Descartes glare points in scattering by icicles: color photographs and a tilted dielectric cylinder model of caustic and glare-point evolution," Applied Optics 37, 1551-1556 (1998).

5. C. M. Mount, D. B. Thiessen, and P. L. Marston, "Scattering observations for tilted transparent fibers: evolution of Airy caustics with cylinder tilt and the caustic merging transition," Applied Optics 37, 1534-1539 (1998).

6. D. S. Langley and P. L. Marston, "Generalized tertiary rainbow of slightly oblate drops: observations with laser illumination," Applied Optics 37, 1520-1526 (1998). [Langley supported by St. John's University.]

7. T. J. Asaki and P. L. Marston, "The effects of a soluble surfactant on quadrupole shape oscillations and dissolution of air bubbles in water," J. Acoust. Soc. Am. 102, 3372-3377 (1997). [Primarily supported by now-expired grant N00014-91-J-1374.]

c. Printed Technical Reports & Non-refereed Papers:

1. P. L. Marston, "Scattering and radiation of high frequency sound in water by elastic objects, particle suspensions, and curved surfaces," Annual Summary Report for ONR Grant N00014-92-J-1600 (issued June, 1997) DTIC Number A328798.

2. K. Gipson, "Leaky Rayleigh wave ultrasonic backscattering enhancements: Experimental tests of theory for tilted solid cylinders and cubes," Ph.D. thesis (Wash. State Univ., 1998) 201 pages.

3. C. M. Mount, "The evolution of the Airy caustics and the caustic merging transition for light scattered from a tilted dielectric cylinder," M.S. thesis (Wash. State Univ., 1998) 41 pages.

4. P. L. Marston, "Approximations for leaky wave amplitudes in acoustic imaging: Applications to high frequency sonar," in *Acoustical Imaging* Vol. 23, S. Lees and L. A. Ferrari, eds., (Plenum Press, New York, 1997) pp. 369-374.

5. P. L. Marston, "A time-resolved glimpse of the terahertz glory," Nature 391, 841-842 (1998).

d. Book chapters published:

1. H. J. Simpson and P. L. Marston, "Parameteric layers, four-wave mixing, and wave-front reversal," in *Nonlinear Acoustics,* M. F. Hamilton and D. T. Blackstock, eds., (Academic Press, 1998) Chapter 14, pp. 399-420.

2. P. L. Marston, "Introduction Chapter–Ultrasonics, Quantum Acoustics, and Physical Effects of Sound," in *Handbook of Acoustics,* M. J. Crocker, editor (John Wiley Press, New York, 1998) Chap. 40, pp. 475-482.

k. Total Number of Graduate Students Supported at Least 25% This Year on This Grant.

Graduate Students: 5

Karen Gipson

Brian T. Hefner

Scot F. Morse

Catherine M. Mount

Florian J. Blonigen (supported on AASERT)

Appendix A

Convolution Formulation for High-Frequency Leaky Wave Scattering Enhancements for Solids and Shells with Truncations: Evaluation of the Surface Integral and Experimental and Computational Tests

Philip L. Marston, Karen Gipson, and Scot F. Morse

Department of Physics, Washington State University, Pullman, WA 99164-2814

Abstract: A convolution formulation has been developed for leaky wave scattering enhancements. Progress is summarized in the testing of this formulation for several situations in which exact solutions are (1) not known (e.g. solid and hollow finite tilted cylinders and retroreflective flat tilted surfaces with corners) or (2) known (e.g. tilted infinite cylinders). Some of the examples considered are for leaky Rayleigh waves on steel objects in water.

INTRODUCTION AND MOTIVATION

The high frequency backscattering from finite targets with truncations can be significantly enhanced at appropriate aspect angles over a wide range of frequencies. The observed contributions from meridional and helical rays reflected from the end of a cylindrical shell as shown in Fig. 1 are examples [1]. The enhancements are typically associated with backscattered wavefronts having a vanishing gaussian curvature which produce a farfield caustic. A formulation has been developed which first approximates the pressure amplitude p_l radiated by the leaky wave at the surface of the scatterer by convolving the local amplitude p_i of the incident wave on the illuminated portion of the surface with a two-dimensional response function [2,3,4]. The resulting two-dimensional integral for p_l at a surface point S may be expressed in the form

$$p_l(S) \approx \int_{\mathcal{D}} p_i(S')\left[\kappa H_0^{(1)}\left(k_p s\right)\right] d\mathcal{A}', \tag{1}$$

where $H_0^{(1)}$ is the Hankel function having an argument proportional to the geodesic distance s between S and an illuminated point at S' where the incident wave complex amplitude is $p_i(S')$ and $d\mathcal{A}'$ is the differential area of the contributing surface patch. The domain $\mathcal{D}$ is restricted depending on the directional properties of the waves of interest such that the integral may be expressed as a convolution of p_i with a response function of limited angular support [2,3]. The leaky wavenumber for the lth class of leaky wave is $k_p = k_l + i\alpha$ where $k_l = kc/c_l$ and k is the wavenumber in water, $\kappa \approx -\alpha k_l \exp(i\varphi_{bl})$, and φ_{bl} is a background phase. For typical applications of interest, the leaky wave damping rate α is sufficiently large that leaky wave reverberations may be neglected so that global resonances are unimportant. The farfield amplitude is then calculated through the evaluation of a Rayleigh propagation integral. We summarize below the status of computational and experimental tests of this formulation.

TILTED INFINITE CIRCULAR CYLINDERS: HELICAL AND MERIDIONAL RAYS

When applied to helical rays on circular cylinders [2], the integral, Eq. (1), was confirmed to recover results from others derived specifically for leaky waves on thin shells. When applied to leaky wave scattering into the meridional plane numerical tests for Rayleigh waves at high frequencies support the use of the approximation [3]. Recently one of us (SFM) confirmed that the partial wave series for thick and thin infinite shells supports the meridional result.

MERIDIONAL RAY END-REFLECTION BACKSCATTERING ENHANCEMENT

The farfield caustic due to the reflected meridional ray in Fig. 1 makes this contribution important when the cylinder tilt γ is close to the leaky wave trace velocity matching angle $\theta_l = \sin^{-1}(c/c_l)$. The surface pressure is approximated using Eq. (1) by introducing a leaky wave amplitude reflection coefficient B at the end of the cylinder and introducing other approximations [3]. The resulting amplitude is geometrically propagated to a plane tangent to the cylinder and a Rayleigh integral is evaluated to give the farfield amplitude. The original result [3] for the case $\gamma = \theta_l$ has recently been extended to describe the degradation of the amplitude when γ is shifted away from θ_l. For scattering by finite cylinders, it is convenient to relate the incident and farfield scattered pressures, p_o and p_{ls}, by a dimensionless form function f_l through the relationship $p_{ls} = (p_o f_l a/2R)\exp(ikR)$ where a is the radius of the cylinder and R is the distance from a reference point on the cylinder. The analysis based on Eq. (1) shows that for leaky waves of interest when ka is large and the tilt $\gamma \approx \theta_l$, typical values of $|f_l|$ are greater than unity. It follows that this elastic contribution to the backscattering amplitude from an appropriately tilted cylinder can be greater than

for reflection from a fixed rigid sphere having the same radius as the cylinder. The confirmation is summarized as follows: *(i) Rayleigh waves on a solid stainless steel cylinder:* One of us (KG) measured the tilt dependence of the backscattering amplitude for a cylinder with a = 19.05 mm and a length L = 254 mm at frequencies of 0.62 and 1.03 MHz where the prediction for the Rayleigh wave [3] is that $\theta_l \approx 30.7°$. The measurements showed that $|f_l|$ is peaked at an angle offset from θ_l due to an O(L/R) geometric angular shift. The observed maximum in $|f_l|_{exp}$ of 2.5 and 3.2 at ka of 50 and 83 were close to the predictions $|f_l|$ = 3.4 and 3.2, respectively, where $|B|$ is 0.34 in the theory. The measurement at ka = 50 may have been affected by interference from weaker helical wave contributions. The results for the angular width also support the theory. *(ii) a_0 leaky Lamb waves on empty cylindrical shells:* These meridional backscattering enhancements demonstrated for finite cylinders in [1] were studied quantitatively with tone burst measurements and with an approximate partial-wave formulation by one of us (SFM). The ray theory is supported in the ka range where $|B| \approx 1$ and may also be useful when $|B| < 1$.

LEAKY WAVE LAUNCHED DIAGONALLY ACROSS A TILTED CYLINDER'S FLAT END

A large enhancement was also observed when the aforementioned stainless steel cylinder was tilted so as to launch a Rayleigh wave which propagated across the end-diagonal in the meridional plane and reflected from the edge as shown in Fig. 2. The backscattering amplitude was predicted by first evaluating a one-dimensional approximation of Eq. (1) to estimate the reflected amplitude p_l with a reflection coefficient B. The curvature of the reflected leaky wavefront was approximated so as to give an estimate for the peak contribution to $|f_l|$. The simplest approximation to $|f_l|$ gave 2.2 and 3.4 at ka of 50 and 83 which were 95% ± 15% of the corresponding observed magnitudes. The procedure for obtaining the estimates appears to be useful for many purposes.

LEAKY WAVE BACKSCATTERING ENHANCEMENT FOR A TILTED SOLID CUBE

Figure 3 shows a leaky wave mechanism for producing a flat backscattered wavefront from a tilted cube when only one of the cube's three euler angles is constrained to lie in a narrow range. For a randomly oriented cube (or certain other square-cornered objects) this becomes the most likely cause of large high frequency backscattering. Calculations based on a one-dimensional approximation to Eq. (1) for the radiated near-field amplitude resulted in the prediction of a large farfield form function. Measurements by one of us (KG) confirmed the general magnitude of the prediction in the case of an appropriately tilted stainless steel cube.

Acknowledgment: This research was supported by the Office of Naval Research.

REFERENCES

1. Morse, S. F., Marston, P. L., and Kaduchak, G., *J. Acoust. Soc. Am.* **103**, 785-794 (1998).
2. Marston, P. L., *J. Acoust. Soc. Am.* **102**, 1628-1638 (1997).
3. Marston, P. L., *J. Acoust. Soc. Am.* **102**, 358-369 (1997), errata at press.
4. Marston, P. L., in *Acoustical Imaging 23rd Symposium*, (Plenum, N.Y., 1998) pp. 369-374.

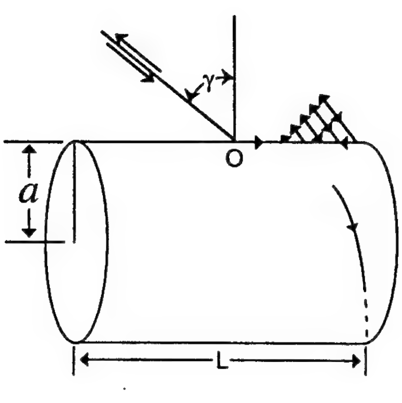

FIGURE 1. Meridional and helical rays.

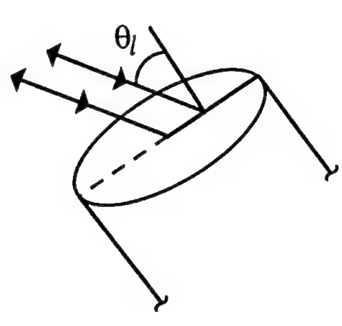

FIGURE 2. Diagonal end ray.

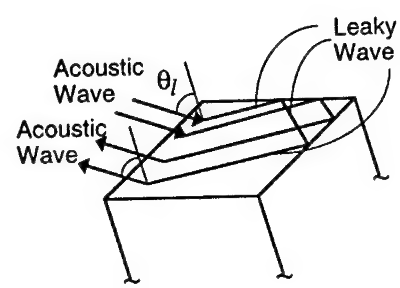

FIGURE 3. Corner retroreflection.

Appendix B

Meridional and Helical Ray Contributions to Backscattering by Tilted Cylindrical Shells: High Frequency Tone Burst and Wide Bandwidth Measurements and Interpretation

Scot F. Morse and Philip L. Marston

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Abstract: When viewed in the frequency-angle domain, the backscattering by truncated thick cylindrical shells in water exhibit various ridges of high backscatter. These enhancements are associated with leaky waves excited on the shell and can be described in terms of meridional and helical rays. The enhancements are not present for a rigid scatterer of the same shape and may be important for interpreting high frequency sonar echoes. The results of broadband measurements are reviewed and tone burst measurements, which isolate the meridional ray feature, are presented. The general angular width and amplitude of the meridional ray enhancement are explained by an approximate ray theory.

INTRODUCTION

The measured global response of backscattering from a thick finite cylindrical shell immersed in water is displayed in Fig. 1(b). A discussion of the backscattering features shown here is given in a previous paper[1]. One important feature previously identified is the ridge of high backscatter curving upwards from $\gamma = 60°$ to $46°$ between 240 and 400 kHz. This corresponds to a meridional ray end-backscattering enhancement[2] associated with coupling to the generalization of the supersonic a_0 Lamb wave. Diagrammed in Fig. 1(a), a meridional leaky wave is launched when the tilt angle, γ, nears a leaky wave coupling angle, $\theta_l = \sin^{-1}(c/c_l)$, where c_l is the phase velocity of the l^{th} class of leaky wave propagating in the axial direction and c is the speed of sound in the surrounding fluid. The leaky wave travels along the meridian of the cylinder, reflects off the truncation and subsequently radiates in the backward direction. Marston[2] pointed out that the gaussian curvature of this backward directed wavefront vanishes for this mechanism, which correspondingly produces a farfield caustic. This mechanism has also been observed for Rayleigh waves on a solid stainless steel cylinder immersed in water[3]. The observed enhancement mechanism may therefore be of importance in high frequency sonar and imaging systems. It is important to point out that for the leaky waves discussed here (e.g. the a_0 on a thick shell and a Rayleigh wave on a solid steel cylinder) the radiation damping is sufficiently high that multiple lengthwise reflections do not contribute to the backscattering.

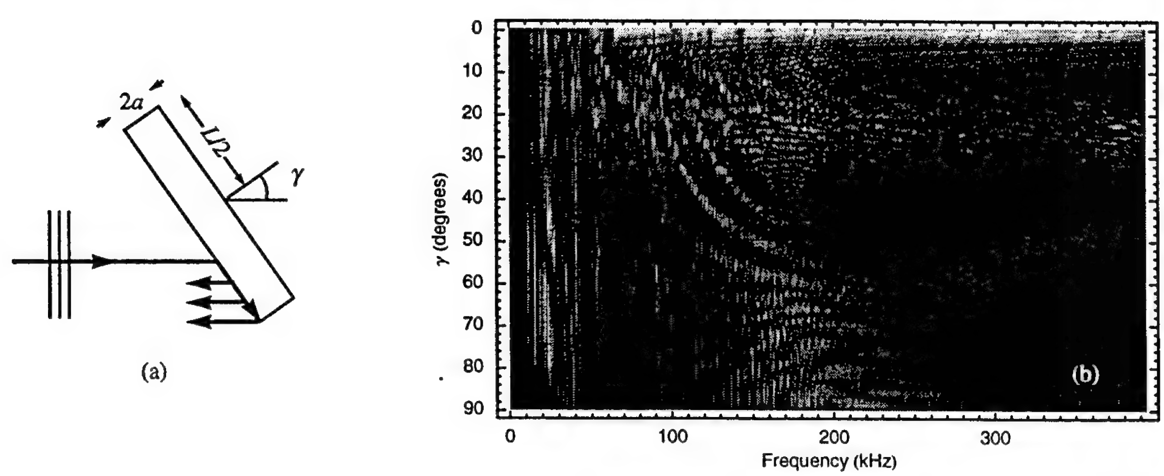

FIGURE 1. (a) Ray diagram for a meridional leaky ray on a finite cylindrical shell. (b) Measured backscattered spectral magnitude of the impulse response for an empty finite cylindrical shell (304 stainless steel) having a thickness to radius ratio of $h/a = 0.076$, an aspect ratio of $L/a = 12$ ($a = 19.05$ mm) and endcaps of thin flat Plexiglas.

MERIDIONAL RAY ENHANCEMENT FOR A SHELL: RAY THEORY AND MEASUREMENTS

A recent extension of the ray analysis of Ref. 2 has enabled an investigation of not only the amplitude of meridional ray enhancements but also their general angular width. Figure 2 shows a comparison of this ray analysis with an approximate partial wave series (PWS) solution[1] for the cylinder of Fig. 1(b) near the observed a_0 meridional ray enhancement at 400 kHz. Both analyses predict a well defined large amplitude feature in the form function centered about the meridional ray coupling angle. In this case the amplitude is predicted to be 5.72 times that for a rigid sphere of the same radius as the cylinder, provided the leaky wave end-reflection coefficient has unit magnitude. (The leaky wave end-reflection coefficient must be specified in the ray formulation but is necessarily equal to unity in the approximate PWS solution.) In the ray analysis the center angle, amplitude and angular width of the enhancement are a function of the leaky wave axial wavenumber $k_p = k_l + i\alpha$ ($\alpha << k_l$, $\alpha L >> 1$). The latter of these conditions assures that the cylinder can be modeled as semi-infinite; consequently, multiple lengthwise reflections of the meridional leaky wave are not included. For the cylinder described here the axial wavenumber was calculated for the zeroth order (azimuthal index) antisymmetric (in the thickness sense) mode of an infinite cylindrical shell using the full elastic solution. In nondimensional form the computed value is $k_p a = 24.43 + i\,0.56$ at $ka = 32.34$, where a is the radius (19.05 mm) and $k = \omega/c$. Recent tone burst experiments by the authors, which isolate the meridional ray enhancement feature in time, have allowed for amplitude measurements as a function of cylinder tilt angle. These measurements yield an enhancement whose maximum value is $|f_{exp}| = 5.68$, which is very similar to that shown in Fig. 2 for the same frequency. The measured FWHM (full-width half-max) is 3.2° compared with the ray theory result of 3.5°. The measured maximum is found at $\gamma = \theta_l = 48.0°$, corresponding to $\mathrm{Re}[k_p a] = 24.03$, while the calculated maximum is at $\gamma = \theta_l = 49.1°$. The difference between the measured and calculated peak angle is easily attributed to inaccurate knowledge of the sound speeds of the cylinder material and is exaggerated by the $\sin^{-1}(k_l/k)$ dependence of the peak angle. By comparing the ray theory result with the experiment the reflection coefficient for the a_0 wave is inferred to be $|R| \approx 5.68/5.72 = 0.99$. Mode conversion upon reflection from the end, resulting in a drop in the reflection coefficient in addition to losses to the surrounding fluid, is not expected to heavily influence the reflection mechanics at this frequency (400 kHz) due to the fact that the threshold of the a_1 mode is estimated to be near 1.1 MHz.

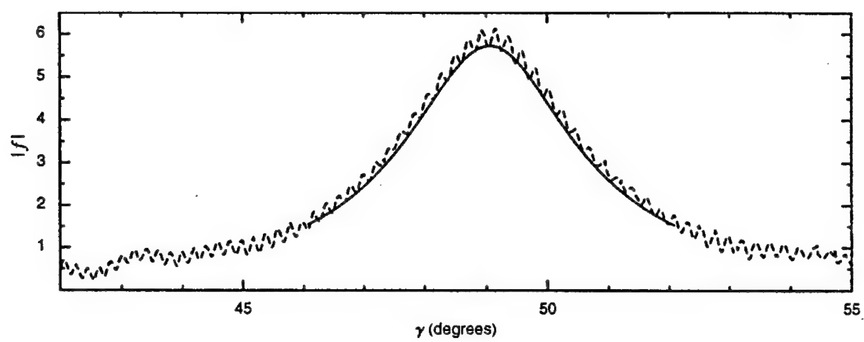

FIGURE 2. Backscattering form function magnitude near the meridional ray enhancement for the a_0 leaky Lamb wave at 400 kHz (ka = 32.34). The dashed curve is an approximate PWS result while the solid curve is the ray theory result. For comparison, backscattering from a rigid sphere of the same radius as the cylinder yields $\left|f_{sph}^{(r)}\right| \approx 1$.

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REFERENCES

1. S. F. Morse, P. L. Marston and G. Kaduchak, *J. Acoust. Soc. Am.* **103**, 785-794 (1998).
2. P. L. Marston, *J. Acoust. Soc. Am.* **102**, 358-369 (1997), errata at press.
3. K. Gipson and P. L. Marston, Washington State University, private communication.

Appendix C

Acoustical Helicoidal Waves and Laguerre-Gaussian Beams: Applications to Scattering and to Angular Momentum Transport

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Abstract: Some properties of acoustical traveling waves with helicoidal or "spiral-like" wavefronts are analyzed. These include orbital angular momentum flux and potentially useful imaging or scattering properties for the discrimination of axisymmetric objects from other objects. Measurements are summarized for a simple single-element ultrasonic transducer for generating approximately helicoidal beam-waves in water.

INTRODUCTION

A potentially useful class of exact and approximate solutions of the Helmholtz equation that do not appear to have been well developed in acoustics are traveling waves having helicoidal wavefronts. The wavefields are cylindrically symmetric except for an azimuthal angular dependence of exp $(im\theta - i\omega t)$ where m is an integer and θ is the azimuthal angle. The amplitude vanishes at a screw-phase dislocation on the z propagation axis [1]. Examples of paraxial helicoidal waves are the Laguerre-Gaussian (LG) beam solutions of the parabolic wave equation which are well known in optics [2]. Linearly polarized electromagnetic LG beams transport angular momentum about the z axis [3] and have been used to apply a torque to optically absorbing spheres, which gives an alternative to torque application based on circular polarization [4]. Since the angular momentum transport is the result of the helicoidal wavefront geometry, the corresponding acoustical LG beam carries an angular momentum analyzed here. Using (r,θ,z) cylindrical coordinates and the standard optical result [2,3], the pressure may be written as the real part of $p(r,\theta,z) = \tilde{p}(r,\theta,z)\exp(ikz - i\omega t)$, where the LG solution of the free-space parabolic equation is

$$\tilde{p}(r,\theta,z) = A_{mn}\left[1+(z/z_R)^2\right]^{-1/2} e^{-(r/w)^2} e^{ikr^2/2R} e^{-i\psi} e^{im\theta}\left((r/w)\sqrt{2}\right)^m L_n^m\left(2r^2/w^2\right), \tag{1}$$

where $w = [2(z_R^2+z^2)/kz_R]^{1/2}$, A_{mn} is a constant, L_n^m is the associated Laguerre polynomial of the indicated argument, $z_R = kw_o^2/2$ is the Rayleigh range, w_o is a beam waist parameter, $R = (z_R^2+z^2)/z$, $\psi = (m+1+2n)\tan^{-1}(z/z_R)$ is a generalized Guoy phase shift, $m = 0, \pm 1, \ldots$, $n = 0, 1, 2\ldots$ affects the number of radial nodes, and z is the distance from the beam waist. The helicoidal wavefront is a consequence of the $\exp(im\theta)$ factor and in the special case of $m = 0$, $R(z)$ take on the significance of the phase-front radius of curvature.

RATIO OF AXIAL ANGULAR MOMENTUM FLUX TO BEAM POWER

Let $\varphi = (ip/\omega\rho)$ denote the complex velocity potential and $\mathbf{v} = -\nabla\mathrm{Re}[\varphi]$ be the fluid velocity where Re denotes the real part. From Eq. (1), the azimuthal velocity is $v_\theta = \mathrm{Re}[-im\varphi/r]$. The average axial angular momentum density of the beam is $\langle(\delta\rho)rv_\theta\rangle$ where $\delta\rho = (c^{-2})\,\mathrm{Re}[p]$ is the first-order change in density due to the acoustic wave and $\langle\ \rangle$ denotes a time average. The angular momentum flux $\langle L_z\rangle$ and power P of the beam are

$$\langle L_z\rangle = 2\pi c\int_0^\infty\langle(\delta\rho)rv_\theta\rangle r\,dr, \quad P = 2\pi\int_0^\infty\langle\mathrm{Re}[-ik\varphi]\,\mathrm{Re}[p]\rangle r\,dr, \tag{2,3}$$

where P is the integral of the local average acoustic intensity $\langle v_z\mathrm{Re}[p]\rangle$. Inspection of Eq. (2) gives $\langle L_z\rangle/P = m/\omega$ for each value of m and n in Eq. (1). This ratio is the same as for an electromagnetic beam [3]. Consequently absorption of acoustic energy from beams with $m \neq 0$ will produce an axial torque on the absorber.

SYMMETRY PROPERTIES AND BACKSCATTERING

For the situations of interest, $m \neq 0$ and p vanishes on the z axis since the L_n^m are regular; for example $m = 1$, $n = 0$ gives $L_n^m = 1$. The symmetry properties of a scatterer with respect to the z axis may be revealed by detecting the backscattering. Suppose, for example, the target is an axisymmetric reflector as in the case of a perpendicular mirror or a spherical reflector located on the axis. For an axisymmetric receiver transducer with a transfer function containing a unimodular phase factor $s(\theta)$ the complex output voltage contains of factor proportional to

$$V = \int_0^{2\pi} s(\theta) e^{im\theta} d\theta, \tag{4}$$

where the exp(imθ) is due to the illumination and the target symmetry. V vanishes if either s is constant (as in a spatially flat transducer) or if the source transducer is used as a receiver so that from reciprocity s = exp(imθ). If s is set to detect a wave of opposite pitch as the transmitted wave, s = exp(-imθ) and V is maximized.

HELICOIDAL TRANSDUCER

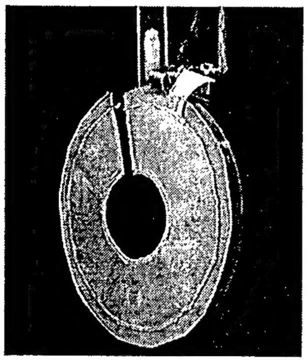
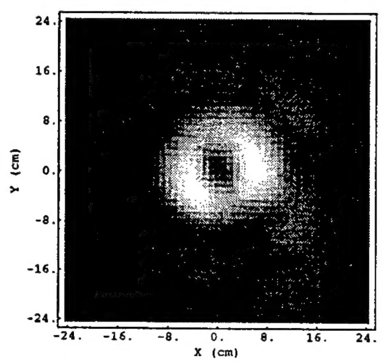


FIGURE 1. Transducer

The transducer used to generate acoustical heliciodal beams is shown in Fig. 1. It is composed of an annular sheet of PVDF attached at its outer edge to a ring of marine brass. The active portion of the PVDF has an outer radius of a = 4.5 cm and an inner radius of b = 1.9 cm. The ring was cut at a point near the transducer's support such that the ring could be twisted like the coil of a spring. By placing a cut through the PVDF correponding to the break in the ring, the ring can be deformed such that the height of the surface of the transducer can be described by $z_T = \lambda\ \theta / 2\ \pi$, where λ is the wavelength of the beam. This should produce a phase-front which has a θ-dependence similar to that of a LG beam with m=1.

The transducer was suspended in a 8' x 8' tank of water and driven at a frequency of 300 kHz. Using a positioning system, it was possible to sample the acoustical signal from the transducer in the plane perpendicular to the face of the transducer at z = 74.6 cm. The intensity distribution in this plane with samples taken using an Edo-sphere hydrophone at 0.5 cm increments is plotted in Fig. 2. In the center of the beam is an approximate null which would be expected as the phase becomes indeterminate at that point. This can also be seen by examining the phase of the beam directly as in Fig. 3(b). The phase of a LG beam with a beam waist, w_o, equal to the outer diameter of the transducer is shown in Fig. 3(a) for comparison. Although the beam generated by the transducer does not have the same uniform structure, it does have a similar spiral in the phase distribution. The intensity distribution in Fig. 2 also displays strong similarities to the intensity distribution of the LG beam, although it is not shown here. These similarities indicate that this may be a useful transducer design for the generation and reception of helicoidal waves.



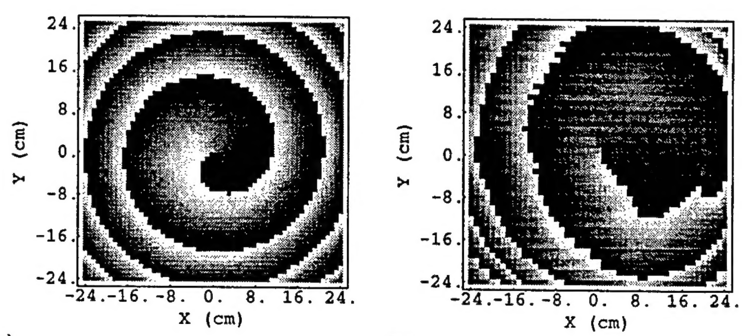


FIGURE 2. Intensity distribution of 300kHz beam at z = 74.6 cm. The central dark spot is low intensity.

FIGURE 3. (a) Phase distribution of a 300kHz LG beam at z = 74.6 cm and w_0 = 4.5 cm (b) Phase distribution for beam from transducer. Dark-to-light indicates a phase increment of 360°.

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REFERENCES

1. Nye, J. F. and Berry, M. V., *Proc. R. Soc. Lond. A* **336**, 165-190 (1974).
2. Siegman, A. E., *Lasers*, (University Science Books, 1986).
3. Allen, L. et al., *Phys. Rev. A* **45**, 8185-8189 (1992).
4. Marston, P. L. and Crichton, J. H., *Phys. Rev. A* **30**, 2508-2516 (1984)

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